The $K^*K\pi$ and $\rho\pi\pi$ couplings in QCD

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Abstract

The light cone QCD sum rules are derived for the $K^*K\pi$ coupling $g_{K^*K\pi}$ and the $\rho\pi\pi$ coupling $g_{\rho\pi\pi}$. The contribution from the excited states and the continuum is subtracted cleanly through the double Borel transform with respect to the two external momenta, p_1^2, $p_2^2 = (p-q)^2$. Our result $g_{K^*K\pi} = (8.7 \pm 0.5)$ and $g_{\rho\pi\pi} = (11.5 \pm 0.8)$ is in good agreement with the experimental value.

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1 Introduction

The $\rho\pi\pi$ ($K^*K\pi$) coupling $g_{\rho\pi\pi}$ ($g_{K^*K\pi}$) plays a very important role in the phenomenological models for the nuclear force and nuclear matter. Now it is widely believed that QCD is the underlying theory of the strong interaction. Yet the complicated infrared behavior of QCD causes the first principle derivation of hadron properties highly nontrivial. So a quantitative calculation of the $g_{\rho\pi\pi}$ ($g_{K^*K\pi}$) coupling with a tractable and reliable theoretical approach proves valuable.

The method of QCD sum rules (QSR), as proposed originally by Shifman, Vainshtein, and Zakharov [1] and adopted, or extended, by many others [2, 3, 4], are very useful in extracting the low-lying hadron masses and couplings. In the QCD sum rule approach the nonperturbative QCD effects are partly taken into account through various condensates in the nontrivial QCD vacuum. In this work we shall use the light cone QCD sum rules (LCQSR) to calculate the $K^*K\pi$ and $\rho\pi\pi$ couplings.

The LCQSR is based on the OPE on the light cone, which is the expansion over the twists of the operators. The main contribution comes from the lowest twist operator. Matrix elements of nonlocal operators sandwiched between a hadronic state and the vacuum defines the hadron wave functions. When the LCQSR is used to calculate the coupling constant, the double Borel transformation is always invoked so that the excited states and the continuum contribution can be treated quite nicely. Moreover, the final sum rule depends only on the value of the hadron wave function at a specific point, which is much better known than the whole wave function [5]. In the present case our sum rules invole with the pion wave function (PWF) $\varphi_\pi(u_0 = \frac{1}{2})$. Note this parameter is universal

in all processes at a given scale. In this respect, $\varphi_\pi(u_0 = \frac{1}{2})$ is a fundamental quantity like the quark condensate. In [6] the value is estimated as $\varphi_\pi(u_0 = \frac{1}{2}) = 1.2 \pm 0.3$ using the pion nucleon coupling constant and the phenomenological $\rho\omega\pi$ coupling constant as inputs. Recently the light cone sum rule for $g_{\pi NN}(q^2 = 0)$ [6] was reanalyzed [7]. The contribution from the gluon condensate $< g_s^2 G^2 >$ and the quark gluon mixed condensate $\langle g_c \bar{q}\sigma \cdot Gq\rangle$ is added. The uncertainty due to λ_N is reduced in the numerical analysis with the help of the Ioffe's mass sum rule. A new value $\varphi_\pi(1/2) = 1.5 \pm 0.2$ [7] is obtained using the experimentally precisely known $g_{\pi NN}$ [8].

The LCQSR has been widely used to derive the couplings of pions with heavy mesons in full QCD [5], in the limit of $m_Q \to \infty$ [9] and $1/m_Q$ correction [10], the couplings of pions with heavy baryons [11], the ρ-decay widths of excited heavy mesons [12] and various semileptonic decays of heavy mesons [13] etc.

Our paper is organized as follows: Section 1 is an introduction. We introduce the two point function for the $K^*K\pi$ vertex in section 2. The definitions of the PWFs are presented in section 3. In the following section we present the LCQSR for the $K^*K\pi$ and $\rho\pi\pi$ couplings. A short summary is given in the last section.

2 Two Point Correlation Function for the $K^*K\pi$ coupling

The dominant decay mode is $K^* \to K\pi$ for K^* and $\rho \to \pi\pi$ for ρ. The relevant decay amplitudes are

$$< K^{*0}(p)\pi^-(q)|K^-(p+q) >= -g_{K^*K\pi} q_\mu \epsilon^\mu \tag{1}$$

$$< K^{*-}(p)\pi^0(q)|K^-(p+q) >= -\frac{g_{K^*K\pi}}{\sqrt{2}} q_\mu \epsilon^\mu \tag{2}$$

$$< \rho^-(p)\pi^0(q)|\pi^-(p+q) >= -g_{\rho\pi\pi} q_\mu \epsilon^\mu \tag{3}$$

where ϵ_μ is the polarization vector of the vector meson. The resulting decay width reads

$$\Gamma(K^* \to K\pi) = \frac{(g_{K^*K\pi})^2 |q_\pi|^3}{16\pi m_{K^*}^2} \tag{4}$$

$$\Gamma(\rho \to \pi\pi) = \frac{(g_{\rho\pi\pi})^2 |q_\pi|^3}{24\pi m_\rho^2} \tag{5}$$

With $\Gamma(K^* \to K\pi) = 50.8$MeV and $\Gamma(\rho \to \pi\pi) = 154$MeV [14] one gets $g_{K^*K\pi} = 9.08$ and $g_{\rho\pi\pi} = 12.16$.

To study these couplings, we start with the two-point correlation function:

$$\Pi(p) = i \int d^4x e^{ipx} \langle \pi^-(q)|T[\bar{d}(x)\gamma^\mu s(x), \bar{s}(0)\gamma^\alpha\gamma_5 u(0)]|0\rangle \,. \tag{6}$$

The instanton contributions may invalidate the usual sum rule techniques for the pseudo-scalar current [15]. We use the pseudo-vector currents for K and π throughout this work.

At the phenomenological level the eq.(6) can be expressed as:

$$\Pi(p_1, p_2, q) = g_{K^*K\pi} \frac{f_{K^*} m_{K^*}}{(p^2 - m_{K^*}^2)} \frac{f_K q_\beta p_2^\alpha}{(p_2^2 - m_K^2)} (g^{\mu\beta} - \frac{p^\mu p^\beta}{m_{K^*}^2}) + \cdots \tag{7}$$

with $p_1 = p$, $p_2 = p+q$. The ellipse denotes the continuum and the off-diagonal transition contribution. The decay constants f_K and f_{K^*} are introduced as:

$$\langle K(p)|\bar{s}(0)\gamma_\mu\gamma_5 u(0)|0\rangle = f_K p_\mu \,, \tag{8}$$

$$\langle K^*|\bar{d}(0)\gamma_\mu s(0)|0\rangle = m_{K^*} f_{K^*}\epsilon_\mu\ , \tag{9}$$

where ϵ_μ is the polarization vector of the K^* meson,

After discarding the single pole terms in (7) which will always be eliminated through double Borel transformation later, the polarization operator can be expressed as

$$\Pi(p_1,p_2,q) = g_{K^*K\pi}\frac{f_{K^*}m_{K^*}}{(p^2-m_{K^*}^2)}\frac{f_K p_2^\alpha}{(p_2^2-m_K^2)}[q^\mu + \frac{1}{2}(1-\frac{m_K^2}{m_{K^*}^2})p^\mu]\ . \tag{10}$$

In the following we shall focus on the tensor structure $p^\alpha q^\mu$.

3 The Formalism of LCQSR and Pion Wave Functions

Neglecting the four particle component of the pion wave function, the expression for $\Pi(p_1,p_2,q)$ with the tensor structure at the quark level reads,

$$\Pi(p_1,p_2,q) = -\mathrm{i}\int d^4x e^{ipx}\mathbf{Tr}\{\langle\pi^-(q)|u(0)\bar{d}(x)|0\rangle\gamma^\mu iS_s(x)\gamma^\alpha\gamma_5\}\ . \tag{11}$$

where $iS_s(x)$ is the full strange quark propagator with both perturbative term and contribution from vacuum fields.

$$\begin{aligned} iS_s(x) = \langle 0|T[s(x),\bar{s}(0)]|0\rangle = -i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\frac{\hat{k}+m_s}{(m_s^2-k^2)} \\ -ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dv[\frac{1}{2}\frac{\hat{k}+m_s}{(m_s^2-k^2)^2}G^{\mu\nu}(vx)\sigma_{\mu\nu}+\frac{1}{m_s^2-k^2}vx_\mu G^{\mu\nu}(vx)\gamma_\nu] \\ -\frac{\langle\bar{s}s\rangle}{12}-\frac{x^2}{192}\langle\bar{s}g_s\sigma\cdot Gs\rangle+\cdots\quad . \end{aligned} \tag{12}$$

where we have introduced $\hat{k}\equiv k_\mu\gamma^\mu$, $m_s = 150\mathrm{MeV}$ is the strange quark mass, $D_\mu = \partial_\mu - ig_sA_\mu$.

By the operator expansion on the light-cone the matrix element of the nonlocal operators between the vacuum and pion state defines the two particle pion wave function. Up to twist four the Dirac components of this wave function can be written as [5]:

$$\begin{aligned} <\pi(q)|\bar{d}(x)\gamma_\mu\gamma_5 u(0)|0> &= -if_\pi q_\mu\int_0^1 du\ e^{iuqx}(\varphi_\pi(u)+x^2g_1(u)+\mathcal{O}(x^4)) \\ &+ f_\pi(x_\mu-\frac{x^2q_\mu}{qx})\int_0^1 du\ e^{iuqx}g_2(u)\ , \end{aligned} \tag{13}$$

$$<\pi(q)|\bar{d}(x)i\gamma_5 u(0)|0> = \frac{f_\pi m_\pi^2}{m_u+m_d}\int_0^1 du\ e^{iuqx}\varphi_P(u)\ , \tag{14}$$

$$<\pi(q)|\bar{d}(x)\sigma_{\mu\nu}\gamma_5 u(0)|0> = i(q_\mu x_\nu - q_\nu x_\mu)\frac{f_\pi m_\pi^2}{6(m_u+m_d)}\int_0^1 du\ e^{iuqx}\varphi_\sigma(u)\ . \tag{15}$$

$$\begin{aligned} &<\pi(q)|\bar{d}(x)\sigma_{\alpha\beta}\gamma_5 g_s G_{\mu\nu}(ux)u(0)|0>= \\ &if_{3\pi}[(q_\mu q_\alpha g_{\nu\beta}-q_\nu q_\alpha g_{\mu\beta})-(q_\mu q_\beta g_{\nu\alpha}-q_\nu q_\beta g_{\mu\alpha})]\int\mathcal{D}\alpha_i\ \varphi_{3\pi}(\alpha_i)e^{iqx(\alpha_1+v\alpha_3)}\quad , \end{aligned} \tag{16}$$

$$<\pi(q)|\bar{d}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)u(0)|0>=$$

$$f_\pi\Big[q_\beta\Big(g_{\alpha\mu}-\frac{x_\alpha q_\mu}{q\cdot x}\Big)-q_\alpha\Big(g_{\beta\mu}-\frac{x_\beta q_\mu}{q\cdot x}\Big)\Big]\int\mathcal{D}\alpha_i\varphi_\perp(\alpha_i)e^{iqx(\alpha_1+v\alpha_3)}$$
$$+f_\pi\frac{q_\mu}{q\cdot x}(q_\alpha x_\beta-q_\beta x_\alpha)\int\mathcal{D}\alpha_i\varphi_\parallel(\alpha_i)e^{iqx(\alpha_1+v\alpha_3)} \qquad (17)$$

and

$$<\pi(q)|\bar d(x)\gamma_\mu g_s\tilde G_{\alpha\beta}(vx)u(0)|0>=$$
$$if_\pi\Big[q_\beta\Big(g_{\alpha\mu}-\frac{x_\alpha q_\mu}{q\cdot x}\Big)-q_\alpha\Big(g_{\beta\mu}-\frac{x_\beta q_\mu}{q\cdot x}\Big)\Big]\int\mathcal{D}\alpha_i\tilde\varphi_\perp(\alpha_i)e^{iqx(\alpha_1+v\alpha_3)}$$
$$+if_\pi\frac{q_\mu}{q\cdot x}(q_\alpha x_\beta-q_\beta x_\alpha)\int\mathcal{D}\alpha_i\tilde\varphi_\parallel(\alpha_i)e^{iqx(\alpha_1+v\alpha_3)}\ . \qquad (18)$$

The operator $\tilde G_{\alpha\beta}$ is the dual of $G_{\alpha\beta}$: $\tilde G_{\alpha\beta}=\frac{1}{2}\epsilon_{\alpha\beta\delta\rho}G^{\delta\rho}$; $\mathcal{D}\alpha_i$ is defined as $\mathcal{D}\alpha_i=d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$. Due to the choice of the gauge $x^\mu A_\mu(x)=0$, the path-ordered gauge factor $P\exp\left(ig_s\int_0^1 du x^\mu A_\mu(ux)\right)$ has been omitted. The coefficient in front of the r.h.s. of eqs. (14), (15) can be written in terms of the light quark condensate $<\bar u u>$ using the PCAC relation: $\mu_\pi=\dfrac{m_\pi^2}{m_u+m_d}=-\dfrac{2}{f_\pi^2}<\bar u u>$.

The PWF $\varphi_\pi(u)$ is associated with the leading twist two operator, $g_1(u)$ and $g_2(u)$ correspond to twist four operators, and $\varphi_P(u)$ and $\varphi_\sigma(u)$ to twist three ones. The function $\varphi_{3\pi}$ is of twist three, while all the PWFs appearing in eqs.(17), (18) are of twist four. The PWFs $\varphi(x_i,\mu)$ (μ is the renormalization point) describe the distribution in longitudinal momenta inside the pion, the parameters x_i ($\sum_i x_i=1$) representing the fractions of the longitudinal momentum carried by the quark, the antiquark and gluon.

The wave function normalizations immediately follow from the definitions (13)-(18): $\int_0^1 du\ \varphi_\pi(u)=\int_0^1 du\ \varphi_\sigma(u)=1$, $\int_0^1 du\ g_1(u)=\delta^2/12$, $\int\mathcal{D}\alpha_i\varphi_\perp(\alpha_i)=\int\mathcal{D}\alpha_i\varphi_\parallel(\alpha_i)=0$, $\int\mathcal{D}\alpha_i\tilde\varphi_\perp(\alpha_i)=-\int\mathcal{D}\alpha_i\tilde\varphi_\parallel(\alpha_i)=\delta^2/3$, with the parameter δ defined by the matrix element: $<\pi(q)|\bar d g_s\tilde G_{\alpha\mu}\gamma^\alpha u|0>=i\delta^2 f_\pi q_\mu$.

4 The LCQSR for the $K^*K\pi$ coupling

Expressing (11) with the strange quark propagator and keeping only the tensor structure $p^\alpha q^\mu$, we arrive at:

$$\Pi(p_1,p_2,q)=-if_\pi\int_0^1\frac{du}{m_s^2-(p+uq)^2}\{\varphi_\pi(u)$$
$$+\frac{\frac{m_s}{3}\mu_\pi\varphi_\sigma(u)+4ug_2(u)-4g_1(u)-4G_2(u)}{m_s^2-(p+uq)^2}-\frac{8m_s^2[g_1(u)+G_2(u)]}{[m_s^2-(p+uq)^2]^2}+\cdots\}\ ,$$

where $\mu_\pi=1.65$GeV, $f_\pi=132$MeV, $G_2(u)=-\int_0^u g_2(u)du$, which arises from integration by parts to absorb the factor $1/(q\cdot x)$,

$$\int_0^1\frac{e^{-iuq\cdot x}}{q\cdot x}g_2(u)du=-i\int_0^1 e^{-iuq\cdot x}G_2(u)du-G_2(u)e^{-iuq\cdot x}|_0^1\ , \qquad (19)$$

Note the second term in (19) vanishes due to $G_2(u_0)=0$ at end points $u_0=0,1$.

Making double Borel transformation with the variables p_1^2 and p_2^2 the single-pole terms in (7) are eliminated. The formula reads:

$$\mathcal{B}_1{}^{M_1^2}_{p_1^2}\mathcal{B}_2{}^{M_2^2}_{p_2^2}\frac{\Gamma(n)}{[m^2-(1-u)p_1^2-up_2^2]^n}=(M^2)^{2-n}e^{-\frac{m^2}{M^2}}\delta(u-u_0)\ . \qquad (20)$$

Subtracting the continuum contribution which is modeled by the dispersion integral in the region $s_1, s_2 \geq s_0$, we arrive at:

$$f_K f_{K^*} m_{K^*} g_{K^* K\pi} e^{-\frac{m_K^2+m_{K^*}^2}{2M^2}} = f_\pi e^{-\frac{u_0(1-u_0)q^2+m_s^2}{M^2}} \{M^2(1-e^{-\frac{s_1}{M^2}})\varphi_\pi(u_0)$$
$$+\frac{m_s}{3}\mu_\pi \varphi_\sigma(u_0) + 4u_0 g_2(u_0) - 4g_1(u_0) - 4G_2(u_0) - \frac{4m_s^2}{M^2}[g_1(u_0)+G_2(u_0)]\} \quad , \qquad (21)$$

where s_1 is the continuum threshold, $u_0 = \frac{M_1^2}{M_1^2+M_2^2}$, $M^2 \equiv \frac{M_1^2 M_2^2}{M_1^2+M_2^2}$, M_1^2, M_2^2 are the Borel parameters. Note all the PWFs involved with the vacuum gluon field do not contribute to the tensor structure $p^\alpha q^\beta$, which greatly simplifies the analysis of the sum rule.

Similarly we can obtain the LCQSR for $\rho\pi\pi$ coupling. In our calculation we take the up and down quark current mass to be zero. Then we arrive at the simple sum rule:

$$f_\rho m_\rho g_{\rho\pi\pi} e^{-\frac{m_\pi^2+m_\rho^2}{2M^2}} = \sqrt{2} e^{-\frac{u_0(1-u_0)q^2+m_s^2}{M^2}} \{M^2(1-e^{-\frac{s_2}{M^2}})\varphi_\pi(u_0)$$
$$+4u_0 g_2(u_0) - 4g_1(u_0) - 4G_2(u_0)\} \quad . \qquad (22)$$

The sum rule (21) and (22) appears asymmetric with the different masses for K and K^* mesons at first sight. However, if a sum rule holds well, there should exist a certain interval called the working region of the Borel parameter, $M_i^2 < M_B^2 < M_f^2$. Within this region the sum rule is insensitive to M_B^2 and stays reasonably stable with the variation of M_B. In other words, every point of M_B in the above interval is equally good for the analysis of the sum rule. So long as the working regions for M_1^2 and M_2^2 have some overlapping region, which does occur in our case, we can choose a common value in the overlapping region for both M_1^2 and M_2^2. From the above argument we know such a choice will not alter significantly the final result of the sum rule, i.e., the choice $M_1^2 = M_2^2$ is allowed in the analysis of the LCQSR. Moreover, the symmetric choice of $M_1^2 = M_2^2$ will enable the clean subtraction of the continuum contribution, which is crucial for the numerical analysis of the sum rules. In contrast, the asymmetric choice will lead to the continuum subtraction extremely difficult in [5], which is the operational and technical motivation for the choice of $u_0 = 1/2$. We shall work in the physical limit $q^2 = m_\pi^2 \to 0$ in (21).

The resulting sum rule depends on the PWFs and the integrals of them at the point $u_0 = \frac{1}{2}$. We adopt $\varphi_\pi(u_0) = 1.5 \pm 0.2$ [7]. For the other PWFs we use the results given in [5], $\varphi_\sigma(u_0) = 1.47$, $g_1(u_0) = 0.022\text{GeV}^2$, $g_2(u_0) = 0$ and $G_2(u_0) = 0.02\text{GeV}^2$ at $u_0 = \frac{1}{2}$ at the scale $\mu = 1\text{GeV}$.

The overlap amplitudes f_{K^*} and f_ρ can be determined in a self-consistent manner making use of the corresponding mass sum rules. For example, for the ρ meson [1, 2], we have:

$$m_\rho^2 f_\rho^2 = \frac{1}{\pi^2} e^{\frac{m_\rho^2}{M_B^2}} \{\frac{1}{4}(1+\frac{\alpha_s}{\pi})M_B^4 E_1 - \frac{b}{48} + \frac{\alpha_s}{\pi}\frac{14}{81}a_q^2 \frac{1}{M_B^2}\}, \qquad (23)$$

where $E_1 \equiv 1 - (1+\frac{s_0}{M_B^2})e^{-\frac{s_0}{M_B^2}}$ (with the continuum threshold $s_0 = 1.5\text{GeV}^2$) is the factor used to subtract the continuum contribution. Numerically, $f_\rho = (0.18 \pm 0.02)\text{GeV}$, $f_{K^*} = (0.21 \pm 0.02)\text{GeV}$, $f_K = (0.15 \pm 0.02)\text{GeV}$ [1, 2, 14, 16].

Note that we have used the pseudo-vector interpolating current, which couples strongly to both pseudo-scalar mesons π, K and pseudo-vector mesons $a_1(1260)$, $K_1(1270)$. $a_1(1260)$ is a broad resonance with a full width of $\sim 200\text{MeV}$. In order to eliminate the contamination from $a_1(1260)$ and $K_1(1270)$, we choose the continuum threshold parameter to be

$s_2 \leq (m_{a_1} - \frac{\Gamma_{a_1}}{2})^2 \sim (1.2 \pm 0.1)\mathrm{GeV}^2$ and $s_1 \leq (m_{K_1} - \frac{\Gamma_{K_1}}{2})^2 \sim (1.5 \pm 0.1)\mathrm{GeV}^2$ These values are consistent with the continuum threshold for the sum rules of pseudo-scalar mesons. But they are slightly smaller than the continuum threshold for vector mesons.

The final sum rules are stable with reasonable variation of the Borel mass around $m_{K^*}^2$ and m_ρ^2 respectively after the exponential factor is moved to the left hand side as can be seen from Fig. 1 and Fig. 2. The terms with the strange quark mass m_s contribute about 10% to the whole sum rule (21). Our final result is $g_{K^*K\pi} = (8.7 \pm 0.5)$ and $g_{\rho\pi\pi} = (11.5 \pm 0.8)$ with the central values of f_ρ, f_K, f_{K^*}, which agrees very well with the value extracted from the experimental data $g_{K^*K\pi} = 9.08$ and $g_{\rho\pi\pi} = 12.16$ [14].

Although the coupling of $K^*\rho K$ may be sizeable, the ρK intermediate states contribute to the continuum only because $m_\rho + m_K = 1.27\mathrm{GeV} > \sqrt{s_1}$. In our approach we have invoked quark-hadron duality and modeled the spectral density with $s > s_1$ at the hadronic side with the free parton-like one. So the ρK contribution is subtracted away. On the other hand, the intermediate state $3\pi K$ does lie around m_{K^*} and below the continuum threshold $\sqrt{s_1}$. But due to the strong suppression from the four-body phase space integral, its contribution is negligible. In other words, the K^* pole term dominates the possible intermediate states with same quantum numbers below the continuum threshold s_1 while those intermediate states above s_1 is subtracted away using the quark-hadron duality assumption, which is the corner stone of the QCD sum rules approach.

In summary we have calculated $K^*K\pi$ and $\rho\pi\pi$ couplings using the light cone QCD sum rules. These couplings are related to the values of PWFs at the point $u_0 = \frac{1}{2}$, which are universal in all processes. Our results are in good agreement with the experimental data.

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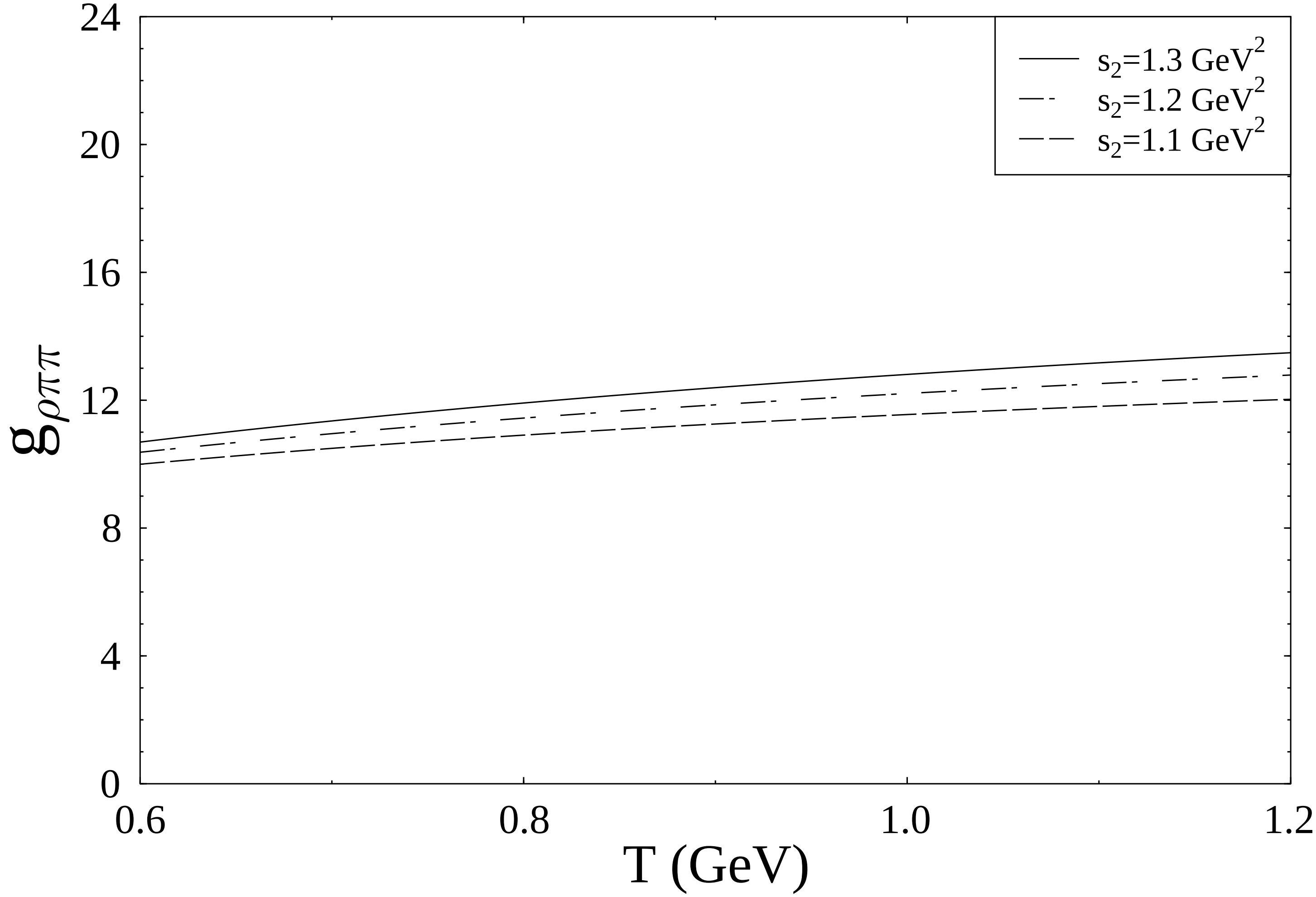
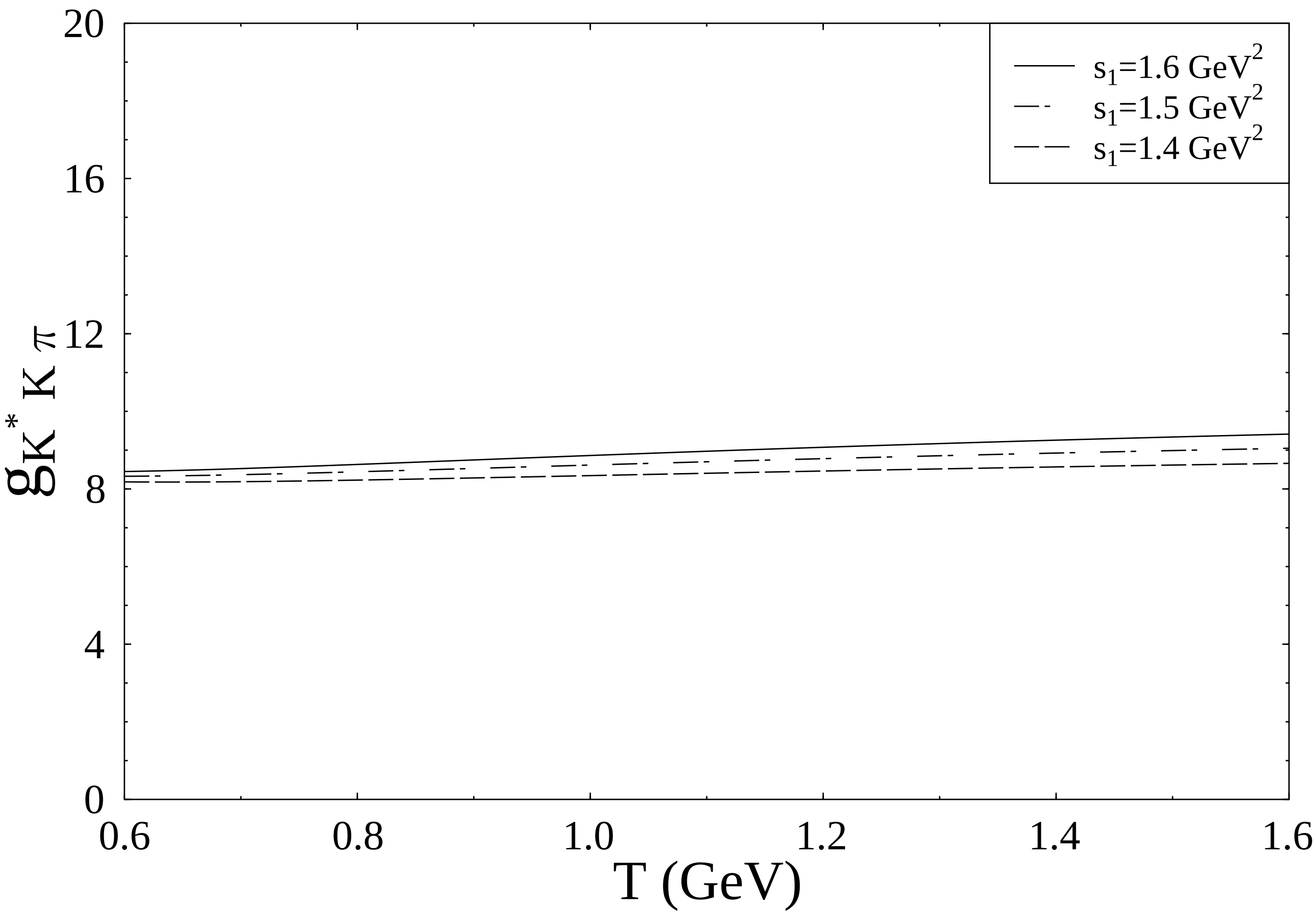
Figure Captions

FIG 1. The sum rule for $g_{\rho\pi\pi}$ as a functions of the Borel parameter M^2. From bottom to top the curves correspond to $s_2 = 1.1, 1.2, 1.3\text{GeV}^2$.

FIG 2. The sum rule for $g_{K^*K\pi}$ as a functions of the Borel parameter M^2. From bottom to top the curves correspond to $s_1 = 1.4, 1.5, 1.6\text{GeV}^2$.



s_2=1.3 GeV^2
s_2=1.2 GeV^2
s_2=1.1 GeV^2
$g_{\rho\pi\pi}$
T (GeV)
0
4
8
12
16
20
24
0.6
0.8
1.0
1.2



s_1=1.6 GeV^2
s_1=1.5 GeV^2
s_1=1.4 GeV^2
20
16
12
8
4
0
$g_{K^* K \pi}$
0.6
0.8
1.0
1.2
1.4
1.6
T (GeV)